norm of operators in intermediate spaces. Here subharmonicity appears in the proof, but the flavor of convexity, which is so characteristic of subharmonicity, appears already in the formulation of the theorem: if T is a linear operator with norms M_0 and M_1 between pairs of spaces (L^{p_0}, L^{q_0}) and (L^{p_1}, L^{q_1}), then for any pair (L^p, L^q) with

$$\frac{1}{p}=\frac{1-\theta}{p_0}+\frac{\theta}{p_1}, \qquad \frac{1}{q}=\frac{1-\theta}{q_0}+\frac{\theta}{q_1}; \qquad 0<\theta<1,$$

the norm of T is at most $M_0^{1-\theta}M_1^{\theta}$, i.e.,

$$\|Tf\|_{L^q}\leqslant M_0^{1-\theta}M_1^{\theta}\,\|f\|_{L^p}.$$

The second application is concerning homogeneous polynomials: it is shown that if $p(z_1, ..., z_n)$ is a homogeneous polynomial of n complex variables, and $E_1, ..., E_n$ are sets with fixed two-dimensional Lebesgue measure $m_1, ..., m_n$, then the minimum of

$$\max\{|p(z_1, ..., z_n)| : z_1 \in E_1, ..., z_n \in E_n\}$$

is attained when all the sets E_j are disks around the origin. The next section contains the important result of Keldysh on the approximability of continuous functions on boundaries of a compact set K by real parts of rational functions (more precisely by functions of the form $\operatorname{Re}\ r(z)+a\log|q(z)|$, where a is a real number and r and q are rational functions with poles resp. poles and zeros from a prescribed set which contains at least one point from every connected component of $\overline{\mathbb{C}}\setminus K$). The next section discusses topics concerning subharmonicity in Banach algebras, while the final section concers complex dynamics such as the capacity and equilibrium measure of Julia sets or Green functions and harmonic measures for attractive basins.

Each chapter contains historical notes, and at the end of the sections there are exercises that direct the reader to further properties or applications. A brief appendix discusses that part of measure theory that is needed to read the text conveniently.

I have thoroughly enjoyed reading this small treatise, and I recommend it for all students and scholars interested in analysis.

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A. I. Stepanets, *Classification and Approximation of Periodic Functions*, Mathematics and Its Applications **333**, Kluwer, Dordrecht, 1995, x + 360 pp.

This book deals with the approximation of periodic functions of one variable by Fourier series. The first chapter opens with a thorough discussion of modulus of continuity in both the uniform and L^p-metrics, and then defines various classes of functions that will be used throughout the book. The author argues that we would like to be able to use all the information we have about the properties of a function in order to understand how well it can be approximated by various means. To do this, we need approximation theorems for quite general classes of functions. So, starting with the well-known classes of functions, the author sets about defining, in a systematic manner, certain classes of functions which become more and more general.

The classic work by Timan [2] (now reprinted by Dover for the benefit of future generations of workers in approximation theory) concludes with a discussion of linear processes of approximation theory. This is the starting point of chapter 2 of the present work by

Stepanets—although I hasten to add that reading Timan [2] is not a prerequisite for reading Stepanets. Suppose that f is Lebesgue integrable over the interval $(0, 2\pi)$. Then we associate with f its Fourier series

$$a_0/2 + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx).$$

Now let $\Lambda = \{\lambda_k^{(n)}: k = 0, 1, ..., n; n = 0, 1, ...\}$ be an infinite triangular matrix of real numbers. We can then consider the linear operator U_n where

$$U_n(f, x, \Lambda) = a_0 \lambda_0^{(n)}/2 + \sum_{k=1}^{n} \lambda_k^{(n)}(a_k \cos kx + b_k \sin kx).$$

Stepanets calls this a "linear method of summation of the Fourier series" of f and he is interested in conditions under which $U_n(f, x, \Lambda)$ converges in some sense to $f(x)$. The aim of this second chapter is to establish formulas for such operators and integral representations for the differences $U_n(f, x, \Lambda) - f(x)$.

Chapter 3 concentrates on the approximation of functions in the spaces C and L^1 by their Fourier series. The author concentrates on establishing best possible estimates for the order of the error $\|S_n(f) - f\|$ where the norm varies with the space and $S_n(f)$ is the nth partial sum of the Fourier series of f. One of the features of this chapter is that the author goes to some length to explain the methods which are behind the proofs of the theorems. In many textbooks, the "tricks of the trade" are just under the surface of the text and the researcher or graduate student who is new to the field discovers these tricks eventually through perseverance. But, at the beginning of this chapter, Stepanets points out some of these methods, gives some simple examples to illustrate them, and only then turns his attention to the main results. Thus, the author's style makes reading related books such as [1, 2] much easier. Chapter 3 concludes with a very interesting section entitled "Behaviour of a sequence of partial Fourier sums near their points of divergence" which is motivated by the famous results of L. Carleson (1966).

In Chapter 4, "Simultaneous approximation of functions and their derivatives by Fourier sums," the author makes substantial use of the classes of functions which are developed and defined in Chapter 1. The present chapter, in particular, would be enhanced by more examples which illustrate the application of these classes of functions.

It is well known that, when one is considering the L^2-norm, the best approximation to a continuous 2π-periodic function f by trigonometric polynomials of order n is provided by the nth partial sum of the Fourier series of f. This classical result is the basis for the development of Chapter 5, which studies the relationship between best approximations and partial sums of Fourier series in various spaces. Stepanets argues from well-known spaces to more general settings and this greatly assists the reader. Chapter 5 mentions a few open problems concerning best constants which connects this work with [1].

The final chapter considers best approximations in the spaces C and L^1 (where the Fourier series does not provide the polynomials of best approximation). Again, the author begins with classical results and develops similar results for more general spaces.

The book is in the "blue series" from Kluwer edited by M. Hazewinkel and the volume is well bound and nicely printed. The quality of the translation by P. V. Malytev and D. V. Malytev is satisfactory. This translation has incorporated revisions and updates of the original Russian edition, which was published in 1987. The bibliography has a strong emphasis on literature written in Russian.

My main criticisms of the book are few. I did not like the notation $x]$ for the integer part of x. The subject index at the back of the book could be more extensive. The discussion of the literature which is confined to "Bibliographic Notes" (pp. 347–350) is too brief for a book of this length on a subject with such a rich history. Apart from these points, I enjoyed reading the book. I would recommend the book for any library which already includes [1] and [2] because the work of Stepanets continues the traditions of these classic works and it is easier to read.

REFERENCES

1. N. P. Korneichuk, "Exact Constants in Approximation Theory," Cambridge Univ. Press, Cambridge, 1991.
2. A. F. Timan, "Theory of Approximation of Functions of a Real Variable," Dover, New York, 1994.

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Proceedings

R. V. M. Zahar, Ed., *Approximation and Computation*: *A Festschrift in Honor of Walter Gautschi*, International Series of Numerical Mathematics, **119**, Birkhauser, Boston, 1994, xlvi + 593 pp.

The 65th birthday of Walter Gautschi provided an opportune moment for an international symposium in his honor. The conference took place in West Lafayette, Indiana, from December 2 to 5, 1993. The main themes were Gautschi's principal interests: approximation, orthogonal polynomials, quadrature, and special functions. Approximately 80 scientists attended the conference, and in this book there are 38 contributions which were all fully refereed. Walter Gautschi wrote a very entertaining personal account, *Reflections and Recollections*, illustrated with some very nice pictures. This is certainly recommended reading. A list of publications of Walter Gautschi containing 150 items is also included.

J. S. Byrnes, J. L. Byrnes, K. A. Hargreaves, and K. Berry, Eds., *Wavelets and Their Applications*, NATO ASI, Series C: Mathematical and Physical Sciences **442**, Kluwer, Dordrecht, 1994, xii + 415 pp.

This volume contains 19 papers presented at the NATO Advanced Study Institute on *Wavelets and Their Applications*, held at Il Ciocco resort near Lucca, Italy, between August 16 and 29, 1992. Many of the world's experts in the field of wavelets were principal speakers. Papers in these proceedings include applications of wavelets to random processes, time-frequency estimation in general and Gabor representations in particular, wavelet packets for data compression, multiscale statistical modeling, applications of frame-like expansions, Clifford wavelets and Hardy spaces (for solving partial differential equations), group representations, the continuous wavelet transform and the generalized modulus of continuity, perturbations of the dilation equation, and neural networks.

S. P. Singh, Ed., *Approximation Theory*, *Wavelets and Applications*, NATO ASI, Series C: Mathematical and Physical Sciences **454**, Kluwer, Dordrecht, 1995, xxiii + 572 pp.

These are the proceedings of the NATO Advanced Study Institute on recent developments in approximation Theory, wavelets, and applications, held at the Hotel Villa del Mare, in Maratea, Italy, from May 16 to 26, 1994. As usual, the proceedings of NATO Advanced Study Institutes contain valuable surveys together with up-to-date developments of the subject. This is usually of great help in giving direction for future research and it stimulates